ALGEBRA I CP and CPE
COURSE CODE: (5110 and 5120)
SUMMER ASSIGNMENT

In order to be successful in high school mathematics, one must have a solid foundation in elementary and pre-algebraic concepts. The following assignment* is intended for students who have been accepted into Algebra I. These topics should have been mastered in middle school. Please review all topics and complete the attached worksheets by coping down the problem onto loose-leaf paper; show all work in a neat and organized manner. We recommend that you periodically go to this packet during the summer rather than attempting to do all of it in your last week. That will allow you to really process these important skills.

This assignment is mandatory and the math department strongly encourages you do this assignment on your own and to the best of your ability. Since the material contained in the summer math packet is *prerequisite material* you are responsible for having learned and retained. If you have forgotten any of these important mathematical concepts, you will find at the end of this assignment, several links to websites that you might find helpful should you have any problems or need some additional support on this assignment.

FINDING EQUIVALENT FRACTIONS AND SIMPLIFYING FRACTIONS

A **fraction** is a number of the form $\frac{a}{b}$ where a is the **numerator** and b is the **denominator**. The value of b cannot be 0. Two fractions that represent the same number are called **equivalent fractions**. To write equivalent fractions, you can divide the numerator and the denominator by the **same** nonzero number.

SAMPLE PROBLEM:

Write the fraction $\frac{10}{15}$ in simplest form.

Divide the numerator and denominator by 5, the greatest common factor of 10 and 15.

$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$ simplest form

Write the fraction in simplest form.

1. $\frac{16}{24}$ **2.** $\frac{3}{12}$ **3.** $\frac{30}{48}$ **4.** $\frac{24}{78}$

5. $\frac{64}{72}$ **6.** $\frac{35}{100}$ **7.** $\frac{21}{81}$ **8.** $\frac{15}{39}$

MIXED NUMBERS AND IMPROPER FRACTIONS

A **mixed number** is the sum of a whole number and a fraction. An **improper fraction** is a fraction with a numerator that is greater than or equal to the denominator. An example of a mixed number is $2\frac{1}{4}$ and the equivalent improper fraction is $\frac{9}{4}$.

SAMPLE PROBLEM:

Write $5\frac{7}{8}$ as an improper fraction.

$5\frac{7}{8} = 5 \overset{+}{} \frac{7}{\times\, 8} = \frac{47}{8}$

Multiply the denominator by the whole number and then add the numerator. This number will be the new numerator and keep the same denominator.

Write the mixed number as an improper fraction.

1. $1\frac{2}{3}$ **2.** $10\frac{3}{10}$ **3.** $2\frac{3}{4}$ **4.** $6\frac{5}{8}$

5. $6\frac{3}{5}$ **6.** $7\frac{2}{9}$ **7.** $12\frac{2}{3}$ **8.** $5\frac{9}{16}$

SAMPLE PROBLEM:

Write $\frac{17}{5}$ as a mixed number.

$$\begin{array}{r} 3 \\ 5\overline{)17} \\ -15 \\ \hline 2 \end{array} \qquad \frac{17}{5} = 3\frac{2}{5}$$

Divide the numerator by the denominator: $17 \div 5$. The quotient is 3 and the remainder is 2.

Write the remainder as a fraction, $\frac{\text{remainder}}{\text{divisor}}$.

Write the improper fraction as a mixed number.

1. $\frac{5}{2}$ **2.** $\frac{37}{3}$ **3.** $\frac{27}{8}$ **4.** $\frac{29}{10}$

5. $\frac{22}{5}$ **6.** $\frac{13}{3}$ **7.** $\frac{43}{9}$ **8.** $\frac{69}{16}$

FRACTION REVIEW: ADDITION & SUBTRACTION

Before fractions can be **added** or **subtracted** they must be presenting "like" quantities. In other words, a ***common denominator*** is required. Knowing your times table facts is incredibly helpful in determining the LCD for each of your problems. This common denominator is the denominator of your result and the numerator is the sum or difference of the numerators. Your result is this fraction in ***simplest*** form. **If mixed numbers are added or subtracted be sure to first rewrite each mixed number as a fraction. It is extremely important that you be able to do all of this work *without* a calculator!**

1. $\frac{5}{8} - \frac{3}{8}$ 2. $\frac{4}{9} - \frac{1}{9}$ 3. $\frac{5}{12} + \frac{3}{12}$

4. $\frac{1}{2} + \frac{1}{8}$ 5. $\frac{3}{5} - \frac{1}{10}$ 6. $\frac{7}{10} + \frac{1}{3}$

7. $\frac{15}{24}-\frac{7}{12}$

8. $5\frac{1}{8}-2\frac{3}{4}$

9. $1\frac{3}{7}+\frac{1}{2}$

10. $4\frac{3}{8}-2\frac{5}{6}$

11. $\frac{3}{7}+\frac{3}{4}$

12. $7\frac{1}{2}+\frac{7}{10}$

13. $5\frac{5}{9}-2\frac{1}{3}$

14. $4\frac{5}{8}-1\frac{3}{16}$

15. $9\frac{2}{5}+3\frac{1}{3}$

16. $6-2\frac{3}{8}$

17. $9\frac{2}{5}+3\frac{1}{2}$

18. $6\frac{5}{7}-2\frac{1}{5}$

19. $\frac{24}{25}-\frac{1}{5}$

20. $5\frac{1}{3}-3\frac{3}{4}$

21. $\frac{9}{10}+\frac{3}{8}$

FRACTION REVIEW: MULTIPLICATION

Good news – no *common denominator* is required here! In fact, in multiplication you can simplify or reduce *before* you actually multiply your numbers. This simplifying or reducing can be done diagonally or vertically – be sure to cancel out a common factor shared by a numerator as well as a denominator. When you do this canceling on the diagonal, you are cross canceling not cross-multiplying. Once simplification is completed, multiply the numerators and multiply the denominators to present your final product. Be sure to convert any mixed number to a fraction before beginning the multiplication procedure. **It is extremely important that you be able to do all of this work *without* a calculator!**

1. $\frac{1}{2}\cdot\frac{1}{2}$

2. $\frac{5}{8}\cdot\frac{4}{15}$

3. $\frac{7}{9}\cdot\frac{1}{5}$

4. $1\frac{2}{3}\cdot\frac{3}{5}$

5. $3\cdot 2\frac{5}{9}$

6. $5\frac{1}{4}\cdot 1\frac{1}{7}$

7. $\frac{5}{9}\cdot 1\frac{1}{2}$

8. $\frac{7}{8}\cdot\frac{4}{9}$

9. $4\frac{1}{4}\cdot\frac{2}{3}$

10. $8\frac{1}{2}\cdot\frac{1}{4}$

11. $\frac{11}{15}\cdot\frac{3}{8}$

FRACTION REVIEW: DIVISION

Good news – since you have now mastered fraction multiplication, division is going to be ***EASY***!!!! Once again, a ***common denominator*** is ***not*** required here! **Division by a fraction converts into multiplication by its reciprocal.** Be sure to change any mixed number to a fraction before beginning this conversion. **It is extremely important that you be able to do all of this work *without* a calculator!**

SAMPLE PROBLEM:

$$2\frac{1}{2} \div 4\frac{1}{6} \rightarrow \frac{5}{2} \div \frac{25}{6} \rightarrow \frac{5}{2} \cdot \frac{6}{25}, cross - cancel, \rightarrow \frac{3}{5}$$

1. $\frac{7}{8} \div \frac{3}{4}$

2. $\frac{5}{12} \div \frac{1}{2}$

3. $\frac{4}{5} \div \frac{2}{3}$

4. $\frac{11}{16} \div 1\frac{1}{2}$

5. $4\frac{1}{2} \div \frac{3}{4}$

6. $2\frac{1}{4} \div 1\frac{1}{3}$

7. $3\frac{2}{5} \div 4$

8. $\frac{3}{10} \div \frac{1}{5}$

9. $\frac{4}{5} \div \frac{1}{2}$

10. $8\frac{4}{5} \div 1\frac{1}{3}$

11. $\frac{12}{13} \div \frac{12}{13}$

12. $\dfrac{24}{\frac{3}{8}}$

13. $\dfrac{\frac{4}{7}}{\frac{4}{5}}$

FRACTIONS, DECIMALS, AND PERCENTS: REVIEW

Percent (%) means "divided by 100." **(a.)** To write a percent as a **decimal**, move the decimal point **two** places to the ***left*** and remove the percent symbol. **(b.)** To write a percent as a fraction in **lowest** terms, first write the percent as a fraction with a denominator of 100. Then simplify if possible. **Keep all reduced fractions as improper fractions.** **It is extremely important that you be able to do all of this work *without* a calculator!**

SAMPLE PROBLEMS:

a.) $85\% = \underset{\leftarrow\leftarrow}{85}\% = 0.85$ **b.)** $85\% = \frac{85}{100} = \frac{17}{20}$

1. 63% 2. 7% 3. 24% 4. 125%

5. 17% 6. 45% 7. 725% 8. 5.2%

9. 62.5% 10. 0.8% 11. 0.12% 12. $33\frac{1}{3}\%$

<u>DECIMALS:</u> (a.) To write a decimal as a percent, move the decimal point **two** places to the ***right*** and add a percent symbol. **(b.)** To write a decimal as a fraction in **lowest** terms, first write the decimal as a fraction with a denominator of 100. Then simplify if possible. **Keep all reduced fractions as improper fractions. It is extremely important that you be able to do all of this work *without* a calculator!**

SAMPLE PROBLEMS:

a.) $0.025 = 0.\underset{\rightarrow\rightarrow}{02}5 = 2.5\%$ **b.)** $0.05 = \frac{5}{100} = \frac{1}{20}$

1. 0.39 2. 0.08 3. 1.5 4. 0.72

5. 2.08 6. 0.02 7. 4.8 8. 3.75

9. 0.85 10. 0.9 11. 0.005 12. 2.01

<u>FRACTIONS</u>: (a.) To write the fractions as a **decimal** divide the denominator into the numerator. **Round** all decimals to the nearest **thousandth**. (That is three decimal places. If needed.) **(b.)** To write a decimal into a **percent**, move the decimal point ***two*** places to the ***right*** and add a percent symbol. **It is extremely important that you be able to do all of this work *without* a calculator!**

SAMPLE PROBLEMS:

a.) $\frac{1}{8} \rightarrow 1 \div 8 = 0.125$ **b.)** $0.125 = 0.\underset{\rightarrow\rightarrow}{12}5 = 12.5\%$

1. $\frac{7}{10}$ 2. $\frac{13}{20}$ 3. $\frac{11}{25}$ 4. $\frac{3}{8}$

5. $\frac{5}{6}$ 6. $\frac{8}{15}$ 7. $\frac{3}{10}$ 8. $\frac{1}{5}$

9. $\frac{1}{20}$ 10. $2\frac{3}{4}$ 11. $3\frac{3}{5}$ 12. $1\frac{5}{12}$

SOLVING PERCENT PROBLEMS

Because **percent** means "**divided by 100**," percents can be written as fractions and percent problems can be solved by means of a **proportion**.

SOLVING Percent Problems Using PROPORTIONS: This approach is particularly helpful when asked to ***find a percent***.

You can represent "***a*** is ***p*** percent of ***b***" using the proportion:

$\frac{a}{b} = \frac{p}{100}$ (or $\frac{is}{of} = \frac{p}{100}$) where ***a*** is a part of the base ***b*** and $\frac{p}{100}$, or ***p%***, is the percent.

SAMPLE PROBLEMS:

a. What percent of 120 is 48?

$$\frac{48}{120} = \frac{p}{100}$$

$$0.4 = \frac{p}{100}$$

$$0.40 = 40\%$$

b. What is 75% of 160?

$$a = 75\% \cdot 160$$

$$a = 0.75 \cdot 160$$

$$a = 120$$

SOLVE each of the following. Be sure to present either a proportion of a percent equation.

1. What percent of 96 is 12?
2. What number is 35% of 18?
3. 14 is 40% of what number?
4. What percent of 125 is 30?
5. What number is 250% of 18?
6. What percent of 58 is 8.7?
7. 30.1 is 35% of what number?
8. What number is 70% of 250?
9. A class of 27 students has 15 girls. What percent of the class is boys?
10. The price of a CD player is $98. What will this CD player cost after a 25% discount?

What will the final cost be after a 7% sales tax is added to the discounted price of this CD player? (Note: tax is calculated on the sales price, not the original price)

11. A i-pod costs $250. What will the i-pod cost after a 20% discount?

12. The TV costs $785. What will the TV cost after the tax of 8% is added?

13. You plan on buying a pair of sunglasses for the summer. The sunglasses cost $135. You have a coupon to get 15% off but you also have to pay the sales tax of 7%. What will be the final cost of these sunglasses?

SCIENTIFIC NOTATION

Numbers such as 1,000,000 and 0.0009 are written in ***standard form***. Another way to write a number is to use ***scientific notation.*** A number is written in ***scientific notation*** when it is of the form $c \times 10^n$ where $1 \le c \le 10$ and n is an integer.

SAMPLE PROBLEMS:

Standard form to scientific notation

a.) $42{,}590{,}000 = 4.259 \times 10^7$
Move decimal point 7 places left.
Exponent is 7.

b.) $0.0000574 = 5.74 \times 10^{-5}$
Move decimal point 5 places to the right.
Exponent is -5.

Write the number in scientific notation.

1. 0.72
2. 0.00406
3. 1,065,250
4. 0.00000007008
5. 2,098,000,000
6. 0.00003402

SAMPLE PROBLEMS:

Scientific notation to standard form

a.) $2.0075 \times 10^6 = 2{,}007{,}500$
Exponent is 6.
Move decimal point 6 places to the right.

b.) $1.685 \times 10^{-4} = 0.0001685$
Exponent is -4.
Move decimal point 4 places to the left.

Write the number in standard form.

1. 3.03×10^4
2. 4.4×10^{-6}
3. 1.2034×10^8
4. 1.544×10^{-5}
5. 5.0023×10^{10}
6. 8.73×10^{-3}

OPPOSITE AND RECIPROCALS

Opposites are two numbers that are the same distance from 0 on a number line but are on opposite sides of 0. The **reciprocal** of a nonzero number *a*, written $\frac{1}{a}$, is also called the **multiplicative inverse** of *a*.

SAMPLE PROBLEM:

a.) -9

opposite: 9

reciprocal: $-\frac{1}{9}$

b.) $\frac{1}{5}$

opposite: $-\frac{1}{5}$

reciprocal: 5

Find the opposite and the reciprocal for each of the following:

1. 3
2. $\frac{2}{7}$
3. -8
4. -2
5. $-\frac{3}{5}$
6. $-\frac{1}{4}$

UNIT RATE

If *a* and *b* are two quantities measured in different units, then the **rate of *a* per *b*** is $\frac{a}{b}$. A **unit rate** is a rate per one unit of a given quantity. To determine a unit rate, write the rate with a denominator of 1. **It is extremely important that you be able to do all of this work *without* a calculator!**

SAMPLE PROBLEM:

A car traveled 648 miles using 18 gallons of gas. Find the unit rate in miles per gallon.

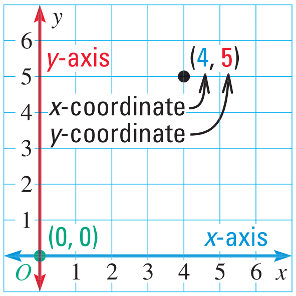
$$\frac{\text{miles}}{\text{gallons}} \rightarrow \frac{648}{18} = \frac{36}{1}$$

The unit rate is 36 miles per gallon.

1. \$90 for 4 tickets.
2. \$51 for 6 hours.
3. 208 miles in 4 hours.
4. 128 ounces for 16 people.
5. \$8.67 for 3 notebooks
6. 65 meters in 3 seconds.

THE COORDINATE PLANE

Just as you use a number line to graph numbers, you use a *coordinate plane* to graph *ordered pair* of numbers. A **coordinate plane** has a horizontal ***x*-axis** and a vertical ***y*-axis** that intersects at a point called the **origin**. The origin is labeled *O*. In an **ordered pair**, the first number is the ***x*-coordinate** and the second number is the ***y*-coordinate**. The coordinates of the origin are (0, 0). The ordered pair (4, 5) is graphed at the bottom.



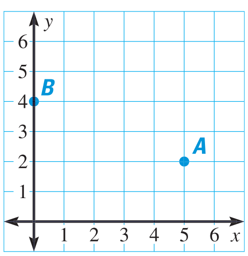
SAMPLE PROBLEM: Give the coordinates of points A and B.

Point A is a 5 units to the right of the origin and 2 units up, so the *x*-coordinate is 5 and the *y*-coordinate is 2.

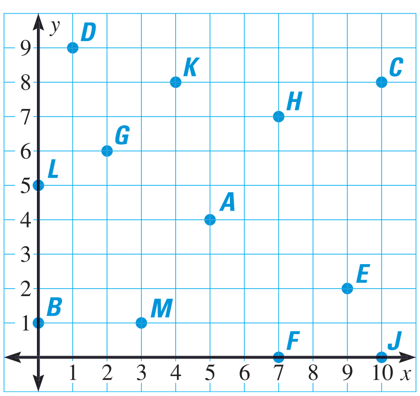
- The coordinates of point A are (5, 2).

Point B is 0 units to the right or left of the origin and 4 units up, to the *x*-coordinate is 0 and the *y*-coordinate is 4.

- The coordinates of point B are (0, 4).



Give the coordinates of the point.



1. A	2. B	3. C	4. D	5. E	6. F
7. G	8. H	9. J	10. K	11. L	12. M

Plot the point in a coordinate plane.

13. M(1, 7)	14. N(2, 1)	15. P(4, 4)	16. Q(0, 3)
17. R(4, 0)	18. S(6, 8)	19. T(3, 6)	20. U(8, 4)
21. V(7, 0)	22. W(0, 8)	23. X(3, 5)	24. Z(5, 6)

PROBLEM SOLVING

One of your primary goals in mathematics should be to become a good problem solver. It will help to approach every problem with an organized plan.

Step 1: Understand the problem. Read the problem. Organize the information you are given and decide what you need to find. Determine whether some of the information given is unnecessary, or whether enough information is given. Supply missing facts, if possible.

Step 2: Make a plan to solve the problem. Choose a strategy. Choose the correct operations. Decide if you will use a tool such as a calculator, a graph, or a spreadsheet.

Step 3: Carry out the plan to solve the problem. Use the strategy and any tools you have chosen. Estimate before you calculate, if possible. Do any calculations that are needed. Answer the questions that the problem asks.

Step 4: Check to see if your answer is reasonable. Reread the problem and see if your answer agrees with the given information.

Problem Solving Strategies:

- Guess, Check, and revise.
- Draw a diagram or a graph.
- Make a table or an organized list.
- Use an equation or a formula.
- Use a proportion.
- Look for a pattern.
- Break the problem into simpler parts.
- Solve a simpler problem.
- Work backwards.

Practice:

1. You bought a magazine for $5 and four erasers. You spent a total of $25. How much did each eraser cost?

2. At a restaurant, Mike and his three friends decided to divide the bill evenly. If each person paid $13 then what was the total bill?

3. If five turkey club sandwiches cost $18.75, how much would seven sandwiches cost?

4. Maria won 40 super bouncy balls playing horseshoes at her school's game night. Later, she gave two to each of her friends. She only has 8 remaining. How many friends does she have?

5. George wants to arrive at school no later than 7:25 A.M. for his first class. It takes him 25 minutes to shower and dress, 15 minutes to eat breakfast, and at least 20 minutes to get to school. What time should he plan to get out of bed?

6. A stray dog ate 12 of your muffins. That was $\frac{3}{10}$ of all of them. How many did you start with?

7. How old am I if 400 reduced by 2 times my age is 244?

8. Carl has \$135 in the bank and plans to save \$5 per week. Jean has \$90 in the bank and plans to save \$10 per week. How many weeks will it be before Jean has at least as much in the bank as Carl?

VARIABALES IN ALGEBRA

A **variable** is a letter that is used to represent one or more numbers. An **algebraic expression**, or *variable expression*, is an expression that includes at least one variable. To *evaluate an algebraic expression*, substitute a number for each variable, perform the operation(s), and simplify the result, if necessary. **No calculator!**

SAMPLE PROBLEM:

Evaluate the expression: $15x$ when $x = 2$

$$15 \cdot x = 15 \cdot 2 = 30$$

Evaluate the expressions when *x* = 3.

1. $7x$
2. $\frac{12}{x}$
3. $x + 9$
4. $20 - x$
5. $\frac{x}{15}$
6. $16 + x$
7. $x - 2$
8. $\frac{5}{6} + x$
9. $\frac{3}{4}x$

EXPONENTS AND POWERS

An expression like 4^6 is called a **power**. The **exponent** 6 represents the number of times the **base** 4 is used as a factor.

$$\underbrace{4^6}_{\text{power}} = \underbrace{4\cdot4\cdot4\cdot4\cdot4\cdot4}_{6 \text{ factors of } 4}$$

SAMPLE PROBLEM:

Evaluate the expression x^3 when $x = 5$.

$$\begin{aligned} x^3 &= 5^3 \\ &= 5\cdot5\cdot5 \\ &= 125 \end{aligned}$$

Evaluate the expression or power.

1. 10^2
2. 1^5
3. 3^4
4. $\left(\frac{3}{5}\right)^3$
5. b^4 when b=9
6. 4^n when $n = 5$
7. $(d-3)^2$ when $d = 13$
8. $16 + x^3$ when $x = 2$

EVALUATING EXPRESSIONS

You must apply the correct **Order of Operations** to do this work successfully. Make sure you use the **left-to-right** rule when it applies. **It is extremely important that you be able to do all of this work *without* a calculator!**

1. $15 \div 3 \cdot 6$
2. $3^3 - 12 \div 4$
3. $10^2 \div 4 + 6$
4. $10^2 \div (4+6)$
5. $3 + 7(3.5 \div 5)$
6. $50 \div (6^2 - 11) - 2$
7. $8 + 3(7-4) - (13-9)$
8. $2[3(7-5) + 4(8+2)]$
9. $8 + 2\cdot 3^2 - 3 + 4^2 - 5$
10. $[(5\cdot 2^3) + 8] \div 16$
11. $[(2 + 4\cdot 3) - 8] + 9^2$
12. $\dfrac{2\cdot7 + 5\cdot3}{30-29}$
13. $\dfrac{9\cdot7^2}{5 + 8^2 - 6}$
14. $\dfrac{20 - [4^2 \div (2+14)] + 5}{4^2 - 13}$

SIMPLIFYING EXPRESSIONS

1. $-3+8$
2. $5+(-7)$
3. $-3+(-11)$
4. $-8-5$
5. $-3-(-7)$
6. $-15+4-12$
7. $11-(-6)-7$
8. $\frac{9}{10}-\frac{1}{2}-\frac{1}{5}$
9. $(-6)(-7)$
10. $3(-8)(-2)$
11. $\left(\frac{2}{3}\right)\left(-\frac{1}{4}\right)$
12. $-4\left(-\frac{3}{4}\right)\left(\frac{2}{5}\right)$
13. $-12 \div 42$
14. $\left(-\frac{4}{5}\right)\div(-8)$
15. $\left(\frac{6}{7}\right)\div\left(-\frac{9}{14}\right)$

DISTRIBUTIVE PROPERTY

The equation $3(x+2)=3(x)+3(2)$ illustrates the **distributive property**, which can be used to find the *product* of a number and a sum or difference.

SAMPLE PROBLEMS:

a.) $4(y+3) = 4y+12$

Multiply 4 times *y* and 4 times 3.

b.) $(y+7)y = y^2+7y$

Multiply *y* time *y* and *y* times 7.

Use the distributive property to write an equivalent expression.

1. $8(x+2)$
2. $(n+6)3$
3. $-(2y-5)$
4. $(3-x)2x$
5. $\frac{1}{2}\left(\frac{1}{2}x-4\right)$
6. $(5w-7)(-3w)$
7. $-3g(6+2g)$
8. $(6n-9)\left(-\frac{2}{3}\right)$